

## **Improving Channel Estimation in OFDM System Using Time Domain Channel Estimation for Time Correlated Rayleigh Fading Channel Model**

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**ABSTRACT :** *In 4G and beyond systems, to achieve higher capacity with better performance, Orthogonal Frequency Division Multiplexing (OFDM) is utilized. OFDM removes the deterioration in the channel due to multipath fading. It converts the frequency selective fading channel into flat fading channel. In this paper, improvement in channel estimation of OFDM system is shown in terms of Bit Error Rate (BER), Symbol Error Rate (SER) and Mean Square Error (MSE). This paper also includes the effect of changing number of subcarriers on the channel estimation performance. Improvement is shown between Least Square Error (LSE) estimation, Minimum Mean Square Error (MMSE) estimation and time domain channel estimation techniques i.e. Discrete Fourier Transform (DFT) and Discrete Cosine Transform (DCT) based channel estimation techniques on time correlated Rayleigh fading channel model i.e. Dent channel model using the 16-QAM modulation technique. Time domain channel estimation techniques are showing better performance with minimum complexity than Least Square Error (LSE) estimation and Minimum Mean Square Error (MMSE) estimation.*

**KEYWORDS** –DCT, DFT, LSE, MMSE, OFDM

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### **I. INTRODUCTION**

The next generation of wireless communication demands for high data rate and better performance for high rate audio and video applications. For high rate data streams, a problem due to inefficient utilization of bandwidth and Inter Symbol Interference (ISI) has occurred. Orthogonal Frequency division multiplexing (OFDM) utilizes the frequency spectrum in an efficient manner. It converts the high rate data streams to multiple low rate data streams by allowing the orthogonal subcarriers to overlap each other. It converts the frequency selective channel to flat fading channel. It uses the concept of cyclic prefix to remove the residual ISI [1].

Better reliability and operation in a wireless system can be obtained by using error correcting schemes and techniques utilized for multiple antenna system like space time block codes (STBC) and different diversity techniques [2]. In order to enhance the performance of the above mentioned techniques, Channel state information (CSI) is needed on the transmitter side. The efficient channel estimation technique will provide better performance. Blind channel estimation technique is not suitable for wireless systems. Semi blind channel estimation technique is preferred because it contains some of the information conveyed on the data in the form of training symbols or pilots. They are added in the transmitted signal in block mode because it is suitable for frequency selective channel [2].

Channel estimation can be done in both frequency and time domain. In frequency domain, Least Square Error (LSE) channel estimation is performed. It has less complexity because it does not require any information about the channel except the position of pilots, but it has degraded performance. Performance is improved by using MMSE channel estimation but at the expense of an increase in complexity [3-5], since it uses auto correlation matrices and noise variance of the channel.

Frequency domain channel estimation techniques are performing better but at the expense of high complexity. So to increase the performance even further with less complexity, channel estimation is performed in the time domain. DFT based channel estimation shows better performance with less complexity than frequency domain channel estimation techniques. In this method, frequency domain estimated channel by LSE estimation is converted to time domain by using Inverse Discrete Fourier Transform (IDFT). Then by applying Spline interpolation, the noise is reduced in time domain by removing certain channel impulse response samples which are profoundly impressed by the disturbance. Remaining samples are converted back to the frequency

domain by using DFT. Thus by removing noise in time domain, performance is increased and complexity is reduced because of the use of fast algorithms i.e. FFT and IFFT.

DFT based channel estimation has degraded efficiency for non-integer multipath samples which causes frequency leakage and aliasing. To overcome this problem, another time domain channel estimation technique is used which is DCT based channel estimation. In this method, as in image compression, it will form images of the non-integer sample and removes the discontinuity between two samples. Time correlated Rayleigh fading channel model i.e. the Dent channel model is used for realizing the improvement of channel estimation performance in OFDM system using above mentioned estimation techniques.

The remaining paper is organized as follows. In section II, channel model is described. In section III, channel estimation techniques are given and simulation results are given in section IV. The paper is concluded in section V.

## II. CHANNEL MODEL

In a wireless communication system, a signal is transmitted from transmitter to receiver through a wireless channel. Due to the presence of various obstacles, multipath propagation occurs. The effect of this will give rise to variation in the signal's amplitude and phase causing multipath fading. The Rayleigh fading channel model is one of the multipath fading channel models. It can be modeled by calculating the real and imaginary components of the complex Gaussian channel. But for some cases only amplitude fluctuations of the Rayleigh fading channel model are on our list.

Jakes proposed a deterministic method for modeling time correlated Rayleigh fading channel [6]. This method is also known as the sum of sinusoidal signal model. In this model, a moving receiver receives  $M$  rays of equal strength at uniformly distributed advent angle  $\gamma_m$  in such a way that the  $n^{\text{th}}$  ray will suffer a Doppler shift of  $\delta_m$ . But Jakes model has some problems that the different signals which are arriving at different angles has a high correlation among each other, which is not desirable. Dent introduces the use of Walsh Hadamard Code i.e. orthogonal codes to remove this correlation problem [6]. Another feature which helps to completely remove the correlation is to allocate equal power to every oscillator. This condition is fulfilled by evaluating Jakes model for distinct advent angles.

Taking  $\gamma_m = \left(\frac{2\pi m}{M}\right) - \left(\frac{\pi}{m}\right)$  and  $\delta_m = \left(\frac{\pi m}{M_0}\right)$ , the waveform can be described by:

$$I_l(t) = \sqrt{\frac{2}{M_0}} \sum_{m=1}^{M_0} P_l(m) (\cos \delta_m + j \sin \delta_m) \times (\cos(\omega_n \cos \gamma_m t) + \varphi_m) \quad (1)$$

Where  $l=1, 2, \dots, M_0$ ,  $M_0=M/4$ , and  $\gamma_m, \delta_m$  and  $\varphi_m$  are independent random phases which are uniformly distributed in  $[0, 2\pi)$ .  $P_l(m)$  is the  $l^{\text{th}}$  Walsh Hadamard codeword in  $m$  which satisfies the following condition:

$$\frac{1}{M_0} \sum_{m=1}^{M_0} P_l^*(m) P_j(m) = \begin{cases} 1, & l = j \\ 0, & l \neq j \end{cases} \quad (2)$$

## III. CHANNEL ESTIMATION

In this section, frequency domain channel estimation i.e. LSE and MMSE channel estimation and then time domain channel estimation i.e. DFT and DCT based channel estimation method is explained.

### 1. Least Square Error (LSE) Channel Estimation

In this method, at the receiver side, the information about the position of the pilots in the transmitted data ( $D$ ) is known. With the information of received data ( $Z$ ) and the position of the pilots, estimated channel ( $\hat{H}$ ) can be calculated by minimizing the least square error:

$$\begin{aligned} C(\hat{H}) &= \|Z - D\hat{H}\|^2 \\ &= (Z - D\hat{H})^H (Z - D\hat{H}) \\ &= Z^H Z - Z^H D\hat{H} - \hat{H}^H D^H Z + \hat{H}^H D^H D\hat{H} \end{aligned} \quad (3)$$

Differentiate  $C(\hat{H})$  with respect to  $\hat{H}$  and then equate it to zero as shown:

$$\frac{\partial C(\hat{H})}{\partial \hat{H}} = -2(D^H Z)^* + 2(D^H D\hat{H}) = 0 \quad (4)$$

We have  $D^H D \hat{H} = D^H Z$  which gives the solution to the LSE channel estimation given by:

$$\hat{h}_{LS} = (D^H D)^{-1} D^H Z = D^{-1} Z \quad (5)$$

This method shows poor performance but has less complexity because it does not require any information about the channel.

## 2. Minimum Mean Square Error (MMSE) Channel Estimation

MMSE estimation of the channel H is given by [7]:

$$\hat{H} = R_{HZ} R_{ZZ}^{-1} Z \quad (6)$$

$$\text{Where } R_{HZ} = R_{HH} W^H D^H \quad (7)$$

$$R_{ZZ} = D W R_{HH} W^H D^H + \sigma_n^2 I_N \quad (8)$$

where  $R_{ZZ}$  is the auto covariance matrix of the received signal Z and  $R_{HZ}$  is the cross covariance matrix of channel H and received signal Z.  $\sigma_n^2$  is noise variance.

The estimated channel  $\hat{h}_{MMSE}$  is given as:

$$\hat{h}_{MMSE} = W \hat{H} = W P W^H D^H Z \quad (9)$$

where W matrix is used to switch channels from time domain to frequency domain. The P matrix is given by [7]:

$$P = R_{HH} [(W^H D^H D W)^{-1} \sigma_n^2 + R_{HH}]^{-1} (W^H D^H D W)^{-1} \quad (10)$$

This method has better performance than LSE estimation but due to the use of auto covariance matrix and noise variance values, complexity of the method increases.

## 3. Discrete Fourier Transform (DFT) based Channel Estimation

DFT based channel estimation is a time domain channel estimation technique. It is used to suppress the noise in time domain because energy is concentrated in time domain. The main asset of this method is that it has less complexity than LSE estimation because of the use of fast algorithms i.e. FFT and IFFT. Performance of DFT based estimation is better than both LSE and MMSE estimation [8]. In this method, first the frequency domain channel estimation is done using LSE channel estimation, which is given as:

$$\hat{h}_{fs} = D^{-1} Z \quad (11)$$

Now estimated output is converted to time domain using the M-point IDFT.

$$\hat{H}_{fs} = IDFT[\hat{h}_{fs}] \quad (12)$$

In the time domain, energy is concentrated to only a small number of samples. Due to multipath fading, a lot of samples in the channel which have lesser energy. So only L samples are considered which have a considerable amount of energy than noise [9], so we set out:

$$\hat{H}_{fs} = \begin{cases} IDFT[\hat{h}_{fs}] & 0 \leq m \leq L - 1 \\ 0 & otherwise \end{cases} \quad (13)$$

After IDFT, zero padding is applied to increase the number of samples:

$$\hat{H}_{zp,fs} = \begin{cases} \hat{H}_{fs} & 0 \leq m \leq L - 1 \\ 0 & otherwise \\ \hat{H}_{fs} & M - L \leq m \leq M - 1 \end{cases} \quad (14)$$

Since the channel response beyond L samples have only noise so this part can be cast aside. Only first L samples are considered in DFT based channel estimation:

$$\hat{h}_{fs} = DFT[\hat{H}_{zp,fs}] \quad 0 \leq m \leq M - 1 \quad (15)$$

So DFT based channel estimation gives better performance because noise is removed in time domain and has less complexity with the use of FFT and IFFT.

## 4. Discrete Cosine Transform (DCT) based Channel Estimation

In DFT based channel estimation, when DFT operation is applied to non-integer multiples of multipath delays, there will be a problem of frequency leakage and thus cause aliasing. This is due to the discontinuity

between two non-integer multiples. A windowing technique based DFT method can be used to remove this problem. But by using this method, there occurs a decrease in spectral efficiency [10]. This problem can be lessened by using another time domain channel estimation technique i.e. DCT based channel estimation. As in image compression, DCT employs mirror extension of M-point data sequence which removes the discontinuous edge. In this method, similar to DFT based channel estimation, firstly the channel is estimated by LSE channel estimation. Then DCT operation is performed, is given in [11] and reproduced as follows:

$$\begin{aligned} \hat{H}_{ls} &= DCT[\tilde{h}_{ls}] \\ &= g_j \sum_{i=0}^{L-1} \tilde{h}_{ls} \cos \frac{\pi(2i+1)j}{2L}, j = 0, 1 \dots L-1 \end{aligned} \quad (16)$$

Where  $g_j = \frac{1}{\sqrt{L}}, j = 0$ ;  $g_j = \sqrt{\frac{2}{L}}, j \neq 0$

In the next step the zeros are inserted at the end of  $\hat{H}_{ls}$  in the DCT domain:

$$\hat{H}_{zp,ls} = \begin{cases} \hat{H}_{ls} & j \leq L-1 \\ 0 & otherwise \end{cases} \quad (17)$$

After that extendible IDCT is applied to remove the shift effect produced by DCT in the time domain which is given by [12] and reproduced as follows:

$$\hat{h}_{ls} = \sum_{j=0}^{L-1} g_j \hat{H}_{zp,ls} \cos((\frac{m}{M} + \frac{1}{2L})\pi j), m = 0, 1 \dots M-1 \quad (18)$$

DCT based channel estimation removes the problem arises in DFT based channel estimation and increase the performance even further.

#### IV. SIMULATION RESULTS

In this section, the channel estimation improvement in OFDM system is shown using LSE, MMSE, DFT and DCT for a time correlated Rayleigh fading channel model (Dent channel model). The results are plotted using MATLAB in terms of bit error rate, symbol error rate, mean square error and number of subcarriers. The Dent channel model is simulated on an OFDM system using a 16-QAM modulation technique and 512 point FFT. The simulation parameters are as shown in Table 1.

**Table 1 Simulation Parameters**

S.No.	Parameters	Values
1.	Number of Subcarriers	512
2.	Number of Symbols	100
3.	Number of Pilots	4
4.	Modulation Technique	16-QAM
5.	Channel Model	Dent
6.	Speed(V)	100 KMPH
7.	Central Carrier Frequency ( $f_c$ )	2000 MHz
8.	Symbol Frequency ( $f_s$ )	10 KSPS
9.	Number of Channel Coefficients(M)	16

##### 1. Bit Error Rate (BER) Comparison

Figure 1 shows Comparison of BER performance of LSE estimation and DFT estimation technique. It is clear from the figure 1 that DFT estimation technique shows better performance than LSE estimation technique. At 24 dB SNR, bit error rate of DFT estimation is 8.7% less than LSE.

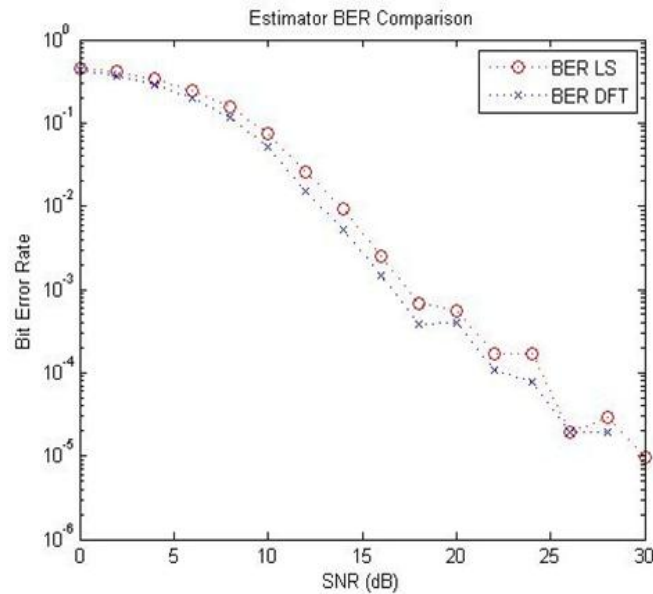


Fig1. BER v/s SNR plot between LSE and DFT estimation techniques

### 2. Symbol Error Rate (SER) Comparison

Figure 2 shows Comparison of SER performance of LSE estimation and DFT estimation technique. It is clear from the figure 2 that DFT estimation technique shows better performance than LSE estimation technique. At 18 dB SNR, symbol error rate of DFT estimation is 7.7% less than LSE estimation.

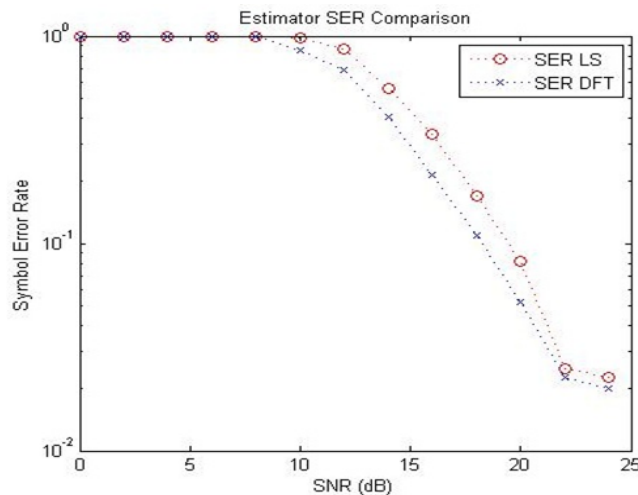


Fig2. SER v/s SNR plot between LSE and DFT estimation techniques

### 3. Mean Square Error (MSE) Comparison

Figure 3 shows the performance comparison of all the channel estimation techniques. It is clear from the figure 3 that DCT based channel estimation is showing better results than DFT based channel estimation. At high SNR, the mean square error of MMSE with DCT increases due to high complexity and mean square error of MMSE with DFT is approximately equal to the LSE estimation with spline interpolator.

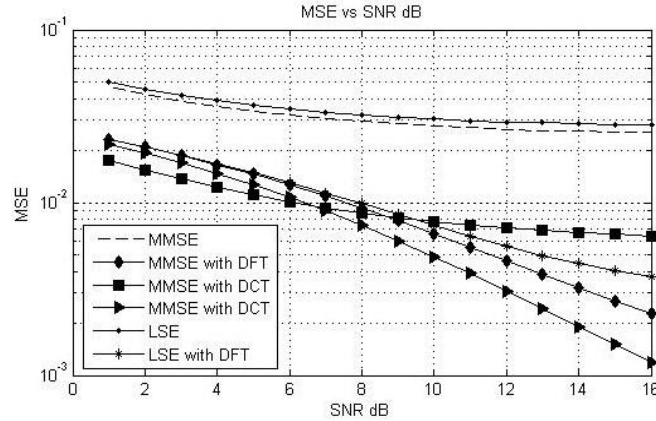


Fig3. MSE v/s SNR plot between all estimation techniques for Dent channel model

#### 4. Number of Subcarriers Comparison

Figure 4 shows the impact of increasing numbers of subcarriers. As the numbers of subcarriers increases, the mean square error starts decreasing that leads to improvement in channel estimation. The results are plotted for the DCT based channel estimation

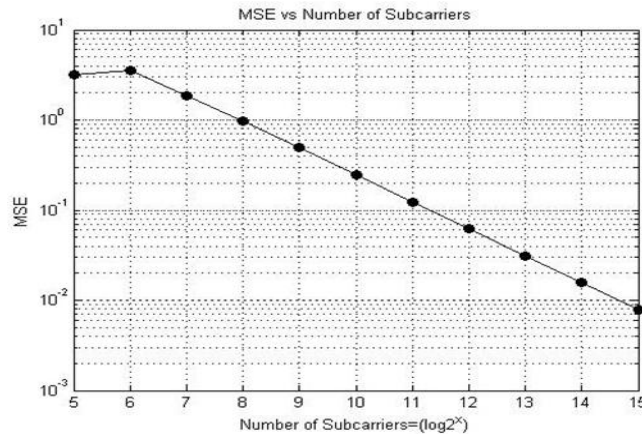


Fig4. MSE v/s Number of Subcarriers plot for DCT based channel estimation technique in OFDM system

### V. CONCLUSION

In this paper, improvement in channel estimation in OFDM system using time domain channel estimation techniques for time correlated Rayleigh fading channel model (Dent channel model) is made. LSE estimation with spline interpolator and MMSE estimation has poor performance and high complexity as compared to time domain channel estimation. DFT based channel estimation technique is applied to the LSE estimated output and reduces the noise in time domain there by increasing the performance and with the use of Fast algorithms like FFT and IFFT, complexity of the estimation is also reduced. For a non - integer number of cycles, there arise the problem of spectral leakage and aliasing while using DFT based channel estimation technique. DCT based channel estimation technique has removed this problem and increases the estimation performance.

Thus DCT based channel estimation technique shows better performance among all the estimation techniques. There is one more aspect of increasing the estimation performance by increasing the number of subcarriers. In this paper, we have realized the improvement of estimation performance by applying it on time correlated Rayleigh fading channel model i.e. Dent channel model. At high SNR, MMSE with DCT performance decreases due to high complexity and performance of MMSE with DFT and LSE (spline) with DFT is approximately equal. The performance can be improved further by the increasing number of subcarriers.



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